Find the general solution of the system of equations.

1. $x' = y, y' = -2x + 3y$
2. $x' = x + 2y, y' = 3x + 4y$
3. $x' = 2y, y' = -x$
4. $x' = 2x + y, y' = x - 2y$
5. $x' = 2x + y + 1, y' = x - 2y - 2$
6. $x' = 2y + t - 1, y' = x - t + 10$
7. $x' = 5x - y - 5, y' = 3x + 2y + 3t$
8. $x' = 2x + 5y, y' = -3x + 8y, x(0) = 2, y(0) = 1$
9. $x' = 5y + t - 3, y' = -3x - y + 10, x(0) = 0, y(0) = 0$
10. $x' = x + y, y' = -x - y, x(0) = 1, y(0) = 0$

11. Tank A contains 2000 lb of salt dissolved in 1000 gal of water. Tank B contains 1000 lb of salt dissolved in 1000 gal of water. The mixture form tank A is pumped to tank B at the rate of 500 gal per hr, while that from tank B is pumped to tank A at the same rate. Assume that the mixture in each tank is kept uniform by stirring. Let $A(t)$ and $B(t)$ be the amount of salt in tanks A and B after t hours, perspectively.

(a) Determine the salt transfer rates from tank A to tank B and from tank B to tank A.

(b) Draw a two-compartment model for $A(t)$ and $B(t)$.

(c) Show that $A(t)$ and $B(t)$ satisfy the differential equations

$$A' = -0.5A + 0.5B, B' = 0.5A - 0.5B$$

(d) Use the initial conditions $A(0) = 2000$ and $B(0) = 1000$ to solve for $A(t)$ and $B(t)$.

(e) What are the equilibrium values of $A(t)$ and $B(t)$.

12. Tank A contains 100 gal of pure water. Tank B contains 33 lb of salt dissolved in 50 gal of water. Pure water is poured into tank B at the rate of 3.5 gal per min while an equal amount of the mixture is drained from the bottom of tank B. The mixture from tank A is pumped to tank B at the rate of 10 gal per min, while that from tank B is pumped to tank A at the same rate.

Assume that the mixture in each tank is kept uniform by stirring. Let $A(t)$ and $B(t)$ be the amount of salt in tanks A and B after t minutes, respectively.

1. Draw a two-compartment model for $A(t)$ and $B(t)$.

2. Write down the system of differential equations that $A(t)$ and $B(t)$ satisfy.

3. Use the initial conditions $A(0) = 0$ and $B(0) = 33$ to solve for $A(t)$ and $B(t)$.